

Al₃Mn Structure:

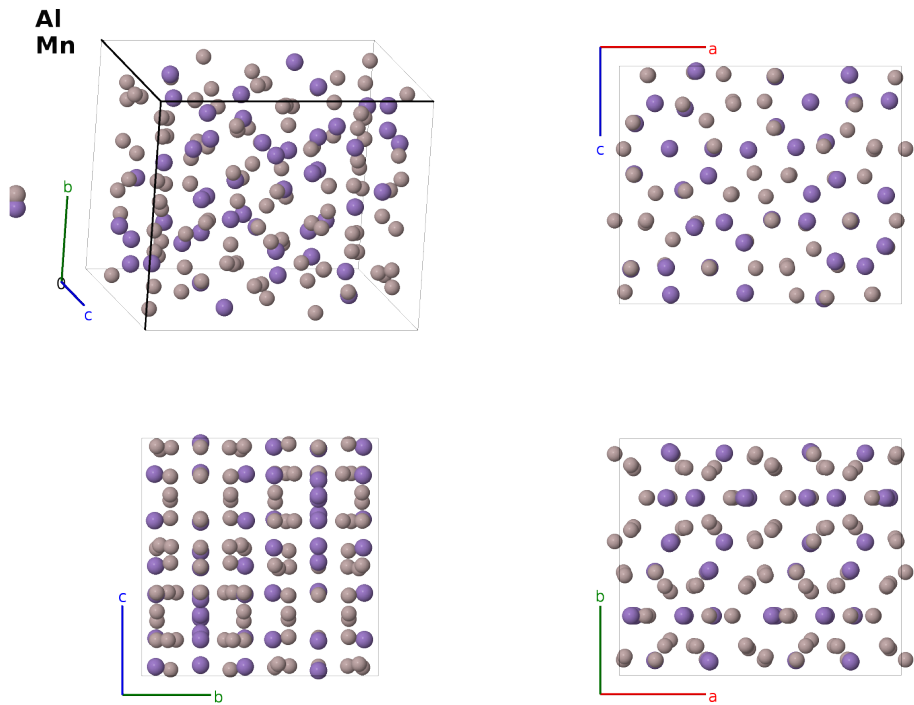
A9B4_oP156_62_5c11d_6c3d-001

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- D. Hicks, M. J. Mehl, M. Esters, C. Oses, O. Levy, G. L. W. Hart, C. Toher, and S. Curtarolo, *The AFLOW Library of Crystallographic Prototypes: Part 3*, Comput. Mater. Sci. **199**, 110450 (2021), doi: 10.1016/j.commatsci.2021.110450.

<https://aflow.org/prototype-encyclopedia/WUDY>

https://aflow.org/prototype-encyclopedia/A9B4_oP156_62_5c11d_6c3d-001



Prototype	Al ₃ Mn
AFLOW prototype label	A9B4_oP156_62_5c11d_6c3d-001
ICSD	105512
CCDC	1663396
Pearson symbol	oP156
Space group number	62
Space group symbol	<i>Pnma</i>
AFLOW prototype command	<pre>aflow --proto=A9B4_oP156_62_5c11d_6c3d-001 --params=a,b/a,c/a,x₁,z₁,x₂,z₂,x₃,z₃,x₄,z₄,x₅,z₅,x₆,z₆,x₇,z₇,x₈,z₈,x₉,z₉,x₁₀, z₁₀,x₁₁,z₁₁,x₁₂,y₁₂,z₁₂,x₁₃,y₁₃,z₁₃,x₁₄,y₁₄,z₁₄,x₁₅,y₁₅,z₁₅,x₁₆,y₁₆,z₁₆,x₁₇,y₁₇,z₁₇,x₁₈, y₁₈,z₁₈,x₁₉,y₁₉,z₁₉,x₂₀,y₂₀,z₂₀,x₂₁,y₂₁,z₂₁,x₂₂,y₂₂,z₂₂,x₂₃,y₂₃,z₂₃,x₂₄,y₂₄,z₂₄,x₂₅,y₂₅, z₂₅</pre>

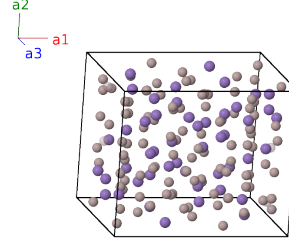
- While all sites are fully occupied, there is some mixing of aluminum and manganese atoms:
 - The Mn-V site is 70% Mn and 30% Al,
 - The Mn-VI site is 80% Mn and 20% Al,
 - The Al-VI site is 10% Mn and 90% Al,
 - The Mn-VIII site is 50% Mn and 50% Al, and
 - The Mn-IX site is 60% Mn and 40% Al.
- This gives a final composition of $\text{Al}_{2.94}\text{Mn}$.

Simple Orthorhombic primitive vectors

$$\mathbf{a}_1 = a \hat{\mathbf{x}}$$

$$\mathbf{a}_2 = b \hat{\mathbf{y}}$$

$$\mathbf{a}_3 = c \hat{\mathbf{z}}$$



Basis vectors

	Lattice coordinates		Cartesian coordinates	Wyckoff position	Atom type
$\mathbf{B}_1$	$= x_1 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_1 \mathbf{a}_3$	$=$	$ax_1 \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} + cz_1 \hat{\mathbf{z}}$	(4c)	Al I
$\mathbf{B}_2$	$= -\left(x_1 - \frac{1}{2}\right) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + \left(z_1 + \frac{1}{2}\right) \mathbf{a}_3$	$=$	$-a\left(x_1 - \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} + c\left(z_1 + \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al I
$\mathbf{B}_3$	$= -x_1 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_1 \mathbf{a}_3$	$=$	$-ax_1 \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} - cz_1 \hat{\mathbf{z}}$	(4c)	Al I
$\mathbf{B}_4$	$= \left(x_1 + \frac{1}{2}\right) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - \left(z_1 - \frac{1}{2}\right) \mathbf{a}_3$	$=$	$a\left(x_1 + \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} - c\left(z_1 - \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al I
$\mathbf{B}_5$	$= x_2 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_2 \mathbf{a}_3$	$=$	$ax_2 \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} + cz_2 \hat{\mathbf{z}}$	(4c)	Al II
$\mathbf{B}_6$	$= -\left(x_2 - \frac{1}{2}\right) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + \left(z_2 + \frac{1}{2}\right) \mathbf{a}_3$	$=$	$-a\left(x_2 - \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} + c\left(z_2 + \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al II
$\mathbf{B}_7$	$= -x_2 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_2 \mathbf{a}_3$	$=$	$-ax_2 \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} - cz_2 \hat{\mathbf{z}}$	(4c)	Al II
$\mathbf{B}_8$	$= \left(x_2 + \frac{1}{2}\right) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - \left(z_2 - \frac{1}{2}\right) \mathbf{a}_3$	$=$	$a\left(x_2 + \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} - c\left(z_2 - \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al II
$\mathbf{B}_9$	$= x_3 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_3 \mathbf{a}_3$	$=$	$ax_3 \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} + cz_3 \hat{\mathbf{z}}$	(4c)	Al III
$\mathbf{B}_{10}$	$= -\left(x_3 - \frac{1}{2}\right) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + \left(z_3 + \frac{1}{2}\right) \mathbf{a}_3$	$=$	$-a\left(x_3 - \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} + c\left(z_3 + \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al III
$\mathbf{B}_{11}$	$= -x_3 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_3 \mathbf{a}_3$	$=$	$-ax_3 \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} - cz_3 \hat{\mathbf{z}}$	(4c)	Al III
$\mathbf{B}_{12}$	$= \left(x_3 + \frac{1}{2}\right) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - \left(z_3 - \frac{1}{2}\right) \mathbf{a}_3$	$=$	$a\left(x_3 + \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} - c\left(z_3 - \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al III
$\mathbf{B}_{13}$	$= x_4 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_4 \mathbf{a}_3$	$=$	$ax_4 \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} + cz_4 \hat{\mathbf{z}}$	(4c)	Al IV
$\mathbf{B}_{14}$	$= -\left(x_4 - \frac{1}{2}\right) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + \left(z_4 + \frac{1}{2}\right) \mathbf{a}_3$	$=$	$-a\left(x_4 - \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} + c\left(z_4 + \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al IV
$\mathbf{B}_{15}$	$= -x_4 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_4 \mathbf{a}_3$	$=$	$-ax_4 \hat{\mathbf{x}} + \frac{3}{4}b \hat{\mathbf{y}} - cz_4 \hat{\mathbf{z}}$	(4c)	Al IV
$\mathbf{B}_{16}$	$= \left(x_4 + \frac{1}{2}\right) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - \left(z_4 - \frac{1}{2}\right) \mathbf{a}_3$	$=$	$a\left(x_4 + \frac{1}{2}\right) \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} - c\left(z_4 - \frac{1}{2}\right) \hat{\mathbf{z}}$	(4c)	Al IV
$\mathbf{B}_{17}$	$= x_5 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_5 \mathbf{a}_3$	$=$	$ax_5 \hat{\mathbf{x}} + \frac{1}{4}b \hat{\mathbf{y}} + cz_5 \hat{\mathbf{z}}$	(4c)	Al V

$\mathbf{B}_{18} =$	$-(x_5 - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_5 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_5 - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_5 + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Al V
$\mathbf{B}_{19} =$	$-x_5 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_5 \mathbf{a}_3$	$=$	$-ax_5 \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_5 \hat{\mathbf{z}}$	(4c)	Al V
$\mathbf{B}_{20} =$	$(x_5 + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_5 - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_5 + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_5 - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Al V
$\mathbf{B}_{21} =$	$x_6 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_6 \mathbf{a}_3$	$=$	$ax_6 \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} + cz_6 \hat{\mathbf{z}}$	(4c)	Mn I
$\mathbf{B}_{22} =$	$-(x_6 - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_6 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_6 - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_6 + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn I
$\mathbf{B}_{23} =$	$-x_6 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_6 \mathbf{a}_3$	$=$	$-ax_6 \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_6 \hat{\mathbf{z}}$	(4c)	Mn I
$\mathbf{B}_{24} =$	$(x_6 + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_6 - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_6 + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_6 - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn I
$\mathbf{B}_{25} =$	$x_7 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_7 \mathbf{a}_3$	$=$	$ax_7 \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} + cz_7 \hat{\mathbf{z}}$	(4c)	Mn II
$\mathbf{B}_{26} =$	$-(x_7 - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_7 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_7 - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_7 + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn II
$\mathbf{B}_{27} =$	$-x_7 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_7 \mathbf{a}_3$	$=$	$-ax_7 \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_7 \hat{\mathbf{z}}$	(4c)	Mn II
$\mathbf{B}_{28} =$	$(x_7 + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_7 - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_7 + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_7 - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn II
$\mathbf{B}_{29} =$	$x_8 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_8 \mathbf{a}_3$	$=$	$ax_8 \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} + cz_8 \hat{\mathbf{z}}$	(4c)	Mn III
$\mathbf{B}_{30} =$	$-(x_8 - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_8 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_8 - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_8 + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn III
$\mathbf{B}_{31} =$	$-x_8 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_8 \mathbf{a}_3$	$=$	$-ax_8 \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_8 \hat{\mathbf{z}}$	(4c)	Mn III
$\mathbf{B}_{32} =$	$(x_8 + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_8 - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_8 + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_8 - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn III
$\mathbf{B}_{33} =$	$x_9 \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_9 \mathbf{a}_3$	$=$	$ax_9 \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} + cz_9 \hat{\mathbf{z}}$	(4c)	Mn IV
$\mathbf{B}_{34} =$	$-(x_9 - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_9 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_9 - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_9 + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn IV
$\mathbf{B}_{35} =$	$-x_9 \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_9 \mathbf{a}_3$	$=$	$-ax_9 \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_9 \hat{\mathbf{z}}$	(4c)	Mn IV
$\mathbf{B}_{36} =$	$(x_9 + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_9 - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_9 + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_9 - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn IV
$\mathbf{B}_{37} =$	$x_{10} \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_{10} \mathbf{a}_3$	$=$	$ax_{10} \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} + cz_{10} \hat{\mathbf{z}}$	(4c)	Mn V
$\mathbf{B}_{38} =$	$-(x_{10} - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_{10} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_{10} - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_{10} + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn V
$\mathbf{B}_{39} =$	$-x_{10} \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_{10} \mathbf{a}_3$	$=$	$-ax_{10} \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_{10} \hat{\mathbf{z}}$	(4c)	Mn V
$\mathbf{B}_{40} =$	$(x_{10} + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_{10} - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_{10} + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_{10} - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn V
$\mathbf{B}_{41} =$	$x_{11} \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 + z_{11} \mathbf{a}_3$	$=$	$ax_{11} \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} + cz_{11} \hat{\mathbf{z}}$	(4c)	Mn VI
$\mathbf{B}_{42} =$	$-(x_{11} - \frac{1}{2}) \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 + (z_{11} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_{11} - \frac{1}{2}) \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} + c(z_{11} + \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn VI
$\mathbf{B}_{43} =$	$-x_{11} \mathbf{a}_1 + \frac{3}{4} \mathbf{a}_2 - z_{11} \mathbf{a}_3$	$=$	$-ax_{11} \hat{\mathbf{x}} + \frac{3}{4}b\hat{\mathbf{y}} - cz_{11} \hat{\mathbf{z}}$	(4c)	Mn VI
$\mathbf{B}_{44} =$	$(x_{11} + \frac{1}{2}) \mathbf{a}_1 + \frac{1}{4} \mathbf{a}_2 - (z_{11} - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_{11} + \frac{1}{2}) \hat{\mathbf{x}} + \frac{1}{4}b\hat{\mathbf{y}} - c(z_{11} - \frac{1}{2}) \hat{\mathbf{z}}$	(4c)	Mn VI
$\mathbf{B}_{45} =$	$x_{12} \mathbf{a}_1 + y_{12} \mathbf{a}_2 + z_{12} \mathbf{a}_3$	$=$	$ax_{12} \hat{\mathbf{x}} + by_{12} \hat{\mathbf{y}} + cz_{12} \hat{\mathbf{z}}$	(8d)	Al VI
$\mathbf{B}_{46} =$	$-(x_{12} - \frac{1}{2}) \mathbf{a}_1 - y_{12} \mathbf{a}_2 + (z_{12} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_{12} - \frac{1}{2}) \hat{\mathbf{x}} - by_{12} \hat{\mathbf{y}} + c(z_{12} + \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VI
$\mathbf{B}_{47} =$	$-x_{12} \mathbf{a}_1 + (y_{12} + \frac{1}{2}) \mathbf{a}_2 - z_{12} \mathbf{a}_3$	$=$	$-ax_{12} \hat{\mathbf{x}} + b(y_{12} + \frac{1}{2}) \hat{\mathbf{y}} - cz_{12} \hat{\mathbf{z}}$	(8d)	Al VI
$\mathbf{B}_{48} =$	$(x_{12} + \frac{1}{2}) \mathbf{a}_1 - (y_{12} - \frac{1}{2}) \mathbf{a}_2 - (z_{12} - \frac{1}{2}) \mathbf{a}_3$	$=$	$a(x_{12} + \frac{1}{2}) \hat{\mathbf{x}} - b(y_{12} - \frac{1}{2}) \hat{\mathbf{y}} - c(z_{12} - \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VI
$\mathbf{B}_{49} =$	$-x_{12} \mathbf{a}_1 - y_{12} \mathbf{a}_2 - z_{12} \mathbf{a}_3$	$=$	$-ax_{12} \hat{\mathbf{x}} - by_{12} \hat{\mathbf{y}} - cz_{12} \hat{\mathbf{z}}$	(8d)	Al VI

$\mathbf{B}_{50} =$	$(x_{12} + \frac{1}{2}) \mathbf{a}_1 + y_{12} \mathbf{a}_2 -$	$=$	$a(x_{12} + \frac{1}{2}) \hat{\mathbf{x}} + by_{12} \hat{\mathbf{y}} - c(z_{12} - \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VI
	$(z_{12} - \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{51} =$	$x_{12} \mathbf{a}_1 - (y_{12} - \frac{1}{2}) \mathbf{a}_2 + z_{12} \mathbf{a}_3$	$=$	$ax_{12} \hat{\mathbf{x}} - b(y_{12} - \frac{1}{2}) \hat{\mathbf{y}} + cz_{12} \hat{\mathbf{z}}$	(8d)	Al VI
$\mathbf{B}_{52} =$	$-(x_{12} - \frac{1}{2}) \mathbf{a}_1 + (y_{12} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a(x_{12} - \frac{1}{2}) \hat{\mathbf{x}} + b(y_{12} + \frac{1}{2}) \hat{\mathbf{y}} +$	(8d)	Al VI
	$(z_{12} + \frac{1}{2}) \mathbf{a}_3$		$c(z_{12} + \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{53} =$	$x_{13} \mathbf{a}_1 + y_{13} \mathbf{a}_2 + z_{13} \mathbf{a}_3$	$=$	$ax_{13} \hat{\mathbf{x}} + by_{13} \hat{\mathbf{y}} + cz_{13} \hat{\mathbf{z}}$	(8d)	Al VII
$\mathbf{B}_{54} =$	$-(x_{13} - \frac{1}{2}) \mathbf{a}_1 - y_{13} \mathbf{a}_2 +$	$=$	$-a(x_{13} - \frac{1}{2}) \hat{\mathbf{x}} - by_{13} \hat{\mathbf{y}} + c(z_{13} + \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VII
	$(z_{13} + \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{55} =$	$-x_{13} \mathbf{a}_1 + (y_{13} + \frac{1}{2}) \mathbf{a}_2 - z_{13} \mathbf{a}_3$	$=$	$-ax_{13} \hat{\mathbf{x}} + b(y_{13} + \frac{1}{2}) \hat{\mathbf{y}} - cz_{13} \hat{\mathbf{z}}$	(8d)	Al VII
$\mathbf{B}_{56} =$	$(x_{13} + \frac{1}{2}) \mathbf{a}_1 - (y_{13} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a(x_{13} + \frac{1}{2}) \hat{\mathbf{x}} - b(y_{13} - \frac{1}{2}) \hat{\mathbf{y}} -$	(8d)	Al VII
	$(z_{13} - \frac{1}{2}) \mathbf{a}_3$		$c(z_{13} - \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{57} =$	$-x_{13} \mathbf{a}_1 - y_{13} \mathbf{a}_2 - z_{13} \mathbf{a}_3$	$=$	$-ax_{13} \hat{\mathbf{x}} - by_{13} \hat{\mathbf{y}} - cz_{13} \hat{\mathbf{z}}$	(8d)	Al VII
$\mathbf{B}_{58} =$	$(x_{13} + \frac{1}{2}) \mathbf{a}_1 + y_{13} \mathbf{a}_2 -$	$=$	$a(x_{13} + \frac{1}{2}) \hat{\mathbf{x}} + by_{13} \hat{\mathbf{y}} - c(z_{13} - \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VII
	$(z_{13} - \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{59} =$	$x_{13} \mathbf{a}_1 - (y_{13} - \frac{1}{2}) \mathbf{a}_2 + z_{13} \mathbf{a}_3$	$=$	$ax_{13} \hat{\mathbf{x}} - b(y_{13} - \frac{1}{2}) \hat{\mathbf{y}} + cz_{13} \hat{\mathbf{z}}$	(8d)	Al VII
$\mathbf{B}_{60} =$	$-(x_{13} - \frac{1}{2}) \mathbf{a}_1 + (y_{13} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a(x_{13} - \frac{1}{2}) \hat{\mathbf{x}} + b(y_{13} + \frac{1}{2}) \hat{\mathbf{y}} +$	(8d)	Al VII
	$(z_{13} + \frac{1}{2}) \mathbf{a}_3$		$c(z_{13} + \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{61} =$	$x_{14} \mathbf{a}_1 + y_{14} \mathbf{a}_2 + z_{14} \mathbf{a}_3$	$=$	$ax_{14} \hat{\mathbf{x}} + by_{14} \hat{\mathbf{y}} + cz_{14} \hat{\mathbf{z}}$	(8d)	Al VIII
$\mathbf{B}_{62} =$	$-(x_{14} - \frac{1}{2}) \mathbf{a}_1 - y_{14} \mathbf{a}_2 +$	$=$	$-a(x_{14} - \frac{1}{2}) \hat{\mathbf{x}} - by_{14} \hat{\mathbf{y}} + c(z_{14} + \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VIII
	$(z_{14} + \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{63} =$	$-x_{14} \mathbf{a}_1 + (y_{14} + \frac{1}{2}) \mathbf{a}_2 - z_{14} \mathbf{a}_3$	$=$	$-ax_{14} \hat{\mathbf{x}} + b(y_{14} + \frac{1}{2}) \hat{\mathbf{y}} - cz_{14} \hat{\mathbf{z}}$	(8d)	Al VIII
$\mathbf{B}_{64} =$	$(x_{14} + \frac{1}{2}) \mathbf{a}_1 - (y_{14} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a(x_{14} + \frac{1}{2}) \hat{\mathbf{x}} - b(y_{14} - \frac{1}{2}) \hat{\mathbf{y}} -$	(8d)	Al VIII
	$(z_{14} - \frac{1}{2}) \mathbf{a}_3$		$c(z_{14} - \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{65} =$	$-x_{14} \mathbf{a}_1 - y_{14} \mathbf{a}_2 - z_{14} \mathbf{a}_3$	$=$	$-ax_{14} \hat{\mathbf{x}} - by_{14} \hat{\mathbf{y}} - cz_{14} \hat{\mathbf{z}}$	(8d)	Al VIII
$\mathbf{B}_{66} =$	$(x_{14} + \frac{1}{2}) \mathbf{a}_1 + y_{14} \mathbf{a}_2 -$	$=$	$a(x_{14} + \frac{1}{2}) \hat{\mathbf{x}} + by_{14} \hat{\mathbf{y}} - c(z_{14} - \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al VIII
	$(z_{14} - \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{67} =$	$x_{14} \mathbf{a}_1 - (y_{14} - \frac{1}{2}) \mathbf{a}_2 + z_{14} \mathbf{a}_3$	$=$	$ax_{14} \hat{\mathbf{x}} - b(y_{14} - \frac{1}{2}) \hat{\mathbf{y}} + cz_{14} \hat{\mathbf{z}}$	(8d)	Al VIII
$\mathbf{B}_{68} =$	$-(x_{14} - \frac{1}{2}) \mathbf{a}_1 + (y_{14} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a(x_{14} - \frac{1}{2}) \hat{\mathbf{x}} + b(y_{14} + \frac{1}{2}) \hat{\mathbf{y}} +$	(8d)	Al VIII
	$(z_{14} + \frac{1}{2}) \mathbf{a}_3$		$c(z_{14} + \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{69} =$	$x_{15} \mathbf{a}_1 + y_{15} \mathbf{a}_2 + z_{15} \mathbf{a}_3$	$=$	$ax_{15} \hat{\mathbf{x}} + by_{15} \hat{\mathbf{y}} + cz_{15} \hat{\mathbf{z}}$	(8d)	Al IX
$\mathbf{B}_{70} =$	$-(x_{15} - \frac{1}{2}) \mathbf{a}_1 - y_{15} \mathbf{a}_2 +$	$=$	$-a(x_{15} - \frac{1}{2}) \hat{\mathbf{x}} - by_{15} \hat{\mathbf{y}} + c(z_{15} + \frac{1}{2}) \hat{\mathbf{z}}$	(8d)	Al IX
	$(z_{15} + \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{71} =$	$-x_{15} \mathbf{a}_1 + (y_{15} + \frac{1}{2}) \mathbf{a}_2 - z_{15} \mathbf{a}_3$	$=$	$-ax_{15} \hat{\mathbf{x}} + b(y_{15} + \frac{1}{2}) \hat{\mathbf{y}} - cz_{15} \hat{\mathbf{z}}$	(8d)	Al IX

$\mathbf{B}_{80} =$	$(x_{16} + \frac{1}{2}) \mathbf{a}_1 - (y_{16} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a (x_{16} + \frac{1}{2}) \hat{\mathbf{x}} - b (y_{16} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8d)$	Al X
	$(z_{16} - \frac{1}{2}) \mathbf{a}_3$		$c (z_{16} - \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{81} =$	$-x_{16} \mathbf{a}_1 - y_{16} \mathbf{a}_2 - z_{16} \mathbf{a}_3$	$=$	$-ax_{16} \hat{\mathbf{x}} - by_{16} \hat{\mathbf{y}} - cz_{16} \hat{\mathbf{z}}$	$(8d)$	Al X
$\mathbf{B}_{82} =$	$(x_{16} + \frac{1}{2}) \mathbf{a}_1 + y_{16} \mathbf{a}_2 -$	$=$	$a (x_{16} + \frac{1}{2}) \hat{\mathbf{x}} + by_{16} \hat{\mathbf{y}} - c (z_{16} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8d)$	Al X
	$(z_{16} - \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{83} =$	$x_{16} \mathbf{a}_1 - (y_{16} - \frac{1}{2}) \mathbf{a}_2 + z_{16} \mathbf{a}_3$	$=$	$ax_{16} \hat{\mathbf{x}} - b (y_{16} - \frac{1}{2}) \hat{\mathbf{y}} + cz_{16} \hat{\mathbf{z}}$	$(8d)$	Al X
$\mathbf{B}_{84} =$	$-(x_{16} - \frac{1}{2}) \mathbf{a}_1 + (y_{16} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a (x_{16} - \frac{1}{2}) \hat{\mathbf{x}} + b (y_{16} + \frac{1}{2}) \hat{\mathbf{y}} +$	$(8d)$	Al X
	$(z_{16} + \frac{1}{2}) \mathbf{a}_3$		$c (z_{16} + \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{85} =$	$x_{17} \mathbf{a}_1 + y_{17} \mathbf{a}_2 + z_{17} \mathbf{a}_3$	$=$	$ax_{17} \hat{\mathbf{x}} + by_{17} \hat{\mathbf{y}} + cz_{17} \hat{\mathbf{z}}$	$(8d)$	Al XI
$\mathbf{B}_{86} =$	$-(x_{17} - \frac{1}{2}) \mathbf{a}_1 - y_{17} \mathbf{a}_2 +$	$=$	$-a (x_{17} - \frac{1}{2}) \hat{\mathbf{x}} - by_{17} \hat{\mathbf{y}} + c (z_{17} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8d)$	Al XI
	$(z_{17} + \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{87} =$	$-x_{17} \mathbf{a}_1 + (y_{17} + \frac{1}{2}) \mathbf{a}_2 - z_{17} \mathbf{a}_3$	$=$	$-ax_{17} \hat{\mathbf{x}} + b (y_{17} + \frac{1}{2}) \hat{\mathbf{y}} - cz_{17} \hat{\mathbf{z}}$	$(8d)$	Al XI
$\mathbf{B}_{88} =$	$(x_{17} + \frac{1}{2}) \mathbf{a}_1 - (y_{17} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a (x_{17} + \frac{1}{2}) \hat{\mathbf{x}} - b (y_{17} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8d)$	Al XI
	$(z_{17} - \frac{1}{2}) \mathbf{a}_3$		$c (z_{17} - \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{89} =$	$-x_{17} \mathbf{a}_1 - y_{17} \mathbf{a}_2 - z_{17} \mathbf{a}_3$	$=$	$-ax_{17} \hat{\mathbf{x}} - by_{17} \hat{\mathbf{y}} - cz_{17} \hat{\mathbf{z}}$	$(8d)$	Al XI
$\mathbf{B}_{90} =$	$(x_{17} + \frac{1}{2}) \mathbf{a}_1 + y_{17} \mathbf{a}_2 -$	$=$	$a (x_{17} + \frac{1}{2}) \hat{\mathbf{x}} + by_{17} \hat{\mathbf{y}} - c (z_{17} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8d)$	Al XI
	$(z_{17} - \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{91} =$	$x_{17} \mathbf{a}_1 - (y_{17} - \frac{1}{2}) \mathbf{a}_2 + z_{17} \mathbf{a}_3$	$=$	$ax_{17} \hat{\mathbf{x}} - b (y_{17} - \frac{1}{2}) \hat{\mathbf{y}} + cz_{17} \hat{\mathbf{z}}$	$(8d)$	Al XI
$\mathbf{B}_{92} =$	$-(x_{17} - \frac{1}{2}) \mathbf{a}_1 + (y_{17} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a (x_{17} - \frac{1}{2}) \hat{\mathbf{x}} + b (y_{17} + \frac{1}{2}) \hat{\mathbf{y}} +$	$(8d)$	Al XI
	$(z_{17} + \frac{1}{2}) \mathbf{a}_3$		$c (z_{17} + \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{93} =$	$x_{18} \mathbf{a}_1 + y_{18} \mathbf{a}_2 + z_{18} \mathbf{a}_3$	$=$	$ax_{18} \hat{\mathbf{x}} + by_{18} \hat{\mathbf{y}} + cz_{18} \hat{\mathbf{z}}$	$(8d)$	Al XII
$\mathbf{B}_{94} =$	$-(x_{18} - \frac{1}{2}) \mathbf{a}_1 - y_{18} \mathbf{a}_2 +$	$=$	$-a (x_{18} - \frac{1}{2}) \hat{\mathbf{x}} - by_{18} \hat{\mathbf{y}} + c (z_{18} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8d)$	Al XII
	$(z_{18} + \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{95} =$	$-x_{18} \mathbf{a}_1 + (y_{18} + \frac{1}{2}) \mathbf{a}_2 - z_{18} \mathbf{a}_3$	$=$	$-ax_{18} \hat{\mathbf{x}} + b (y_{18} + \frac{1}{2}) \hat{\mathbf{y}} - cz_{18} \hat{\mathbf{z}}$	$(8d)$	Al XII
$\mathbf{B}_{96} =$	$(x_{18} + \frac{1}{2}) \mathbf{a}_1 - (y_{18} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a (x_{18} + \frac{1}{2}) \hat{\mathbf{x}} - b (y_{18} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8d)$	Al XII
	$(z_{18} - \frac{1}{2}) \mathbf{a}_3$		$c (z_{18} - \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{97} =$	$-x_{18} \mathbf{a}_1 - y_{18} \mathbf{a}_2 - z_{18} \mathbf{a}_3$	$=$	$-ax_{18} \hat{\mathbf{x}} - by_{18} \hat{\mathbf{y}} - cz_{18} \hat{\mathbf{z}}$	$(8d)$	Al XII
$\mathbf{B}_{98} =$	$(x_{18} + \frac{1}{2}) \mathbf{a}_1 + y_{18} \mathbf{a}_2 -$	$=$	$a (x_{18} + \frac{1}{2}) \hat{\mathbf{x}} + by_{18} \hat{\mathbf{y}} - c (z_{18} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8d)$	Al XII
	$(z_{18} - \frac{1}{2}) \mathbf{a}_3$				
$\mathbf{B}_{99} =$	$x_{18} \mathbf{a}_1 - (y_{18} - \frac{1}{2}) \mathbf{a}_2 + z_{18} \mathbf{a}_3$	$=$	$ax_{18} \hat{\mathbf{x}} - b (y_{18} - \frac{1}{2}) \hat{\mathbf{y}} + cz_{18} \hat{\mathbf{z}}$	$(8d)$	Al XII
$\mathbf{B}_{100} =$	$-(x_{18} - \frac{1}{2}) \mathbf{a}_1 + (y_{18} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a (x_{18} - \frac{1}{2}) \hat{\mathbf{x}} + b (y_{18} + \frac{1}{2}) \hat{\mathbf{y}} +$	$(8d)$	Al XII
	$(z_{18} + \frac{1}{2}) \mathbf{a}_3$		$c (z_{18} + \frac{1}{2}) \hat{\mathbf{z}}$		
$\mathbf{B}_{101$					

$\mathbf{B}_{110} =$	$-\left(x_{20} - \frac{1}{2}\right) \mathbf{a}_1 - y_{20} \mathbf{a}_2 +$	$=$	$-a\left(x_{20} - \frac{1}{2}\right) \hat{\mathbf{x}} - by_{20} \hat{\mathbf{y}} + c\left(z_{20} + \frac{1}{2}\right) \hat{\mathbf{z}}$	(8d)	Al XIV
	$\left(z_{20} + \frac{1}{2}\right) \mathbf{a}_3$				
$\mathbf{B}_{111} =$	$-x_{20} \mathbf{a}_1 + \left(y_{20} + \frac{1}{2}\right) \mathbf{a}_2 - z_{20} \mathbf{a}_3$	$=$	$-ax_{20} \hat{\mathbf{x}} + b\left(y_{20} + \frac{1}{2}\right) \hat{\mathbf{y}} - cz_{20} \hat{\mathbf{z}}$	(8d)	Al XIV
$\mathbf{B}_{112} =$	$\left(x_{20} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{20} - \frac{1}{2}\right) \mathbf{a}_2 -$	$=$	$a\left(x_{20} + \frac{1}{2}\right) \hat{\mathbf{x}} - b\left(y_{20} - \frac{1}{2}\right) \hat{\mathbf{y}} -$	(8d)	Al XIV
	$\left(z_{20} - \frac{1}{2}\right) \mathbf{a}_3$		$c\left(z_{20} - \frac{1}{2}\right) \hat{\mathbf{z}}$		
$\mathbf{B}_{113} =$	$-x_{20} \mathbf{a}_1 - y_{20} \mathbf{a}_2 - z_{20} \mathbf{a}_3$	$=$	$-ax_{20} \hat{\mathbf{x}} - by_{20} \hat{\mathbf{y}} - cz_{20} \hat{\mathbf{z}}$	(8d)	Al XIV
$\mathbf{B}_{114} =$	$\left(x_{20} + \frac{1}{2}\right) \mathbf{a}_1 + y_{20} \mathbf{a}_2 -$	$=$	$a\left(x_{20} + \frac{1}{2}\right) \hat{\mathbf{x}} + by_{20} \hat{\mathbf{y}} - c\left(z_{20} - \frac{1}{2}\right) \hat{\mathbf{z}}$	(8d)	Al XIV
	$\left(z_{20} - \frac{1}{2}\right) \mathbf{a}_3$				
$\mathbf{B}_{115} =$	$x_{20} \mathbf{a}_1 - \left(y_{20} - \frac{1}{2}\right) \mathbf{a}_2 + z_{20} \mathbf{a}_3$	$=$	$ax_{20} \hat{\mathbf{x}} - b\left(y_{20} - \frac{1}{2}\right) \hat{\mathbf{y}} + cz_{20} \hat{\mathbf{z}}$	(8d)	Al XIV
$\mathbf{B}_{116} =$	$-\left(x_{20} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{20} + \frac{1}{2}\right) \mathbf{a}_2 +$	$=$	$-a\left(x_{20} - \frac{1}{2}\right) \hat{\mathbf{x}} + b\left(y_{20} + \frac{1}{2}\right) \hat{\mathbf{y}} +$	(8d)	Al XIV
	$\left(z_{20} + \frac{1}{2}\right) \mathbf{a}_3$		$c\left(z_{20} + \frac{1}{2}\right) \hat{\mathbf{z}}$		
$\mathbf{B}_{117} =$	$x_{21} \mathbf{a}_1 + y_{21} \mathbf{a}_2 + z_{21} \mathbf{a}_3$	$=$	$ax_{21} \hat{\mathbf{x}} + by_{21} \hat{\mathbf{y}} + cz_{21} \hat{\mathbf{z}}$	(8d)	Al XV
$\mathbf{B}_{118} =$	$-\left(x_{21} - \frac{1}{2}\right) \mathbf{a}_1 - y_{21} \mathbf{a}_2 +$	$=$	$-a\left(x_{21} - \frac{1}{2}\right) \hat{\mathbf{x}} - by_{21} \hat{\mathbf{y}} + c\left(z_{21} + \frac{1}{2}\right) \hat{\mathbf{z}}$	(8d)	Al XV
	$\left(z_{21} + \frac{1}{2}\right) \mathbf{a}_3$				
$\mathbf{B}_{119} =$	$-x_{21} \mathbf{a}_1 + \left(y_{21} + \frac{1}{2}\right) \mathbf{a}_2 - z_{21} \mathbf{a}_3$	$=$	$-ax_{21} \hat{\mathbf{x}} + b\left(y_{21} + \frac{1}{2}\right) \hat{\mathbf{y}} - cz_{21} \hat{\mathbf{z}}$	(8d)	Al XV
$\mathbf{B}_{120} =$	$\left(x_{21} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{21} - \frac{1}{2}\right) \mathbf{a}_2 -$	$=$	$a\left(x_{21} + \frac{1}{2}\right) \hat{\mathbf{x}} - b\left(y_{21} - \frac{1}{2}\right) \hat{\mathbf{y}} -$	(8d)	Al XV
	$\left(z_{21} - \frac{1}{2}\right) \mathbf{a}_3$		$c\left(z_{21} - \frac{1}{2}\right) \hat{\mathbf{z}}$		
$\mathbf{B}_{121} =$	$-x_{21} \mathbf{a}_1 - y_{21} \mathbf{a}_2 - z_{21} \mathbf{a}_3$	$=$	$-ax_{21} \hat{\mathbf{x}} - by_{21} \hat{\mathbf{y}} - cz_{21} \hat{\mathbf{z}}$	(8d)	Al XV
$\mathbf{B}_{122} =$	$\left(x_{21} + \frac{1}{2}\right) \mathbf{a}_1 + y_{21} \mathbf{a}_2 -$	$=$	$a\left(x_{21} + \frac{1}{2}\right) \hat{\mathbf{x}} + by_{21} \hat{\mathbf{y}} - c\left(z_{21} - \frac{1}{2}\right) \hat{\mathbf{z}}$	(8d)	Al XV
	$\left(z_{21} - \frac{1}{2}\right) \mathbf{a}_3$				
$\mathbf{B}_{123} =$	$x_{21} \mathbf{a}_1 - \left(y_{21} - \frac{1}{2}\right) \mathbf{a}_2 + z_{21} \mathbf{a}_3$	$=$	$ax_{21} \hat{\mathbf{x}} - b\left(y_{21} - \frac{1}{2}\right) \hat{\mathbf{y}} + cz_{21} \hat{\mathbf{z}}$	(8d)	Al XV
$\mathbf{B}_{124} =$	$-\left(x_{21} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{21} + \frac{1}{2}\right) \mathbf{a}_2 +$	$=$	$-a\left(x_{21} - \frac{1}{2}\right) \hat{\mathbf{x}} + b\left(y_{21} + \frac{1}{2}\right) \hat{\mathbf{y}} +$	(8d)	Al XV
	$\left(z_{21} + \frac{1}{2}\right) \mathbf{a}_3$		$c\left(z_{21} + \frac{1}{2}\right) \hat{\mathbf{z}}$		
$\mathbf{B}_{125} =$	$x_{22} \mathbf{a}_1 + y_{22} \mathbf{a}_2 + z_{22} \mathbf{a}_3$	$=$	$ax_{22} \hat{\mathbf{x}} + by_{22} \hat{\mathbf{y}} + cz_{22} \hat{\mathbf{z}}$	(8d)	Al XVI
$\mathbf{B}_{126} =$	$-\left(x_{22} - \frac{1}{2}\right) \mathbf{a}_1 - y_{22} \mathbf{a}_2 +$	$=$	$-a\left(x_{22} - \frac{1}{2}\right) \hat{\mathbf{x}} - by_{22} \hat{\mathbf{y}} + c\left(z_{22} + \frac{1}{2}\right) \hat{\mathbf{z}}$	(8d)	Al XVI
	$\left(z_{22} + \frac{1}{2}\right) \mathbf{a}_3$				
$\mathbf{B}_{127} =$	$-x_{22} \mathbf{a}_1 + \left(y_{22} + \frac{1}{2}\right) \mathbf{a}_2 - z_{22} \mathbf{a}_3$	$=$	$-ax_{22} \hat{\mathbf{x}} + b\left(y_{22} + \frac{1}{2}\right) \hat{\mathbf{y}} - cz_{22} \hat{\mathbf{z}}$	(8d)	Al XVI
$\mathbf{B}_{128} =$	$\left(x_{22} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{22} - \frac{1}{2}\right) \mathbf{a}_2 -$	$=$	$a\left(x_{22} + \frac{1}{2}\right) \hat{\mathbf{x}} - b\left(y_{22} - \frac{1}{2}\right) \hat{\mathbf{y}} -$	(8d)	Al XVI
	$\left(z_{22} - \frac{1}{2}\right) \mathbf{a}_3$		$c\left(z_{22} - \frac{1}{2}\right) \hat{\mathbf{z}}$		
$\mathbf{B}_{129} =$	$-x_{22} \mathbf{a}_1 - y_{22} \mathbf{a}_2 - z_{22} \mathbf{a}_3$	$=$	$-ax_{22} \hat{\mathbf{x}} - by_{22} \hat{\mathbf{y}} - cz_{22} \hat{\mathbf{z}}$	(8d)	Al XVI
$\mathbf{B}_{130} =$	$\left(x_{22} + \frac{1}{2}\right) \mathbf{a}_1 + y_{22} \mathbf{a}_2 -$	$=$	$a\left(x_{22} + \frac{1}{2}\right) \hat{\mathbf{x}} + by_{22} \hat{\mathbf{y}} - c\left(z_{22} - \frac{1}{$		

$$\begin{aligned}
\mathbf{B}_{140} &= -\left(x_{23} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{23} + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{23} + \frac{1}{2}\right) \mathbf{a}_3 = -a\left(x_{23} - \frac{1}{2}\right) \hat{\mathbf{x}} + b\left(y_{23} + \frac{1}{2}\right) \hat{\mathbf{y}} + c\left(z_{23} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn VII} \\
\mathbf{B}_{141} &= x_{24} \mathbf{a}_1 + y_{24} \mathbf{a}_2 + z_{24} \mathbf{a}_3 = ax_{24} \hat{\mathbf{x}} + by_{24} \hat{\mathbf{y}} + cz_{24} \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{142} &= -\left(x_{24} - \frac{1}{2}\right) \mathbf{a}_1 - y_{24} \mathbf{a}_2 + \left(z_{24} + \frac{1}{2}\right) \mathbf{a}_3 = -a\left(x_{24} - \frac{1}{2}\right) \hat{\mathbf{x}} - by_{24} \hat{\mathbf{y}} + c\left(z_{24} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{143} &= -x_{24} \mathbf{a}_1 + \left(y_{24} + \frac{1}{2}\right) \mathbf{a}_2 - z_{24} \mathbf{a}_3 = -ax_{24} \hat{\mathbf{x}} + b\left(y_{24} + \frac{1}{2}\right) \hat{\mathbf{y}} - cz_{24} \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{144} &= \left(x_{24} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{24} - \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{24} - \frac{1}{2}\right) \mathbf{a}_3 = a\left(x_{24} + \frac{1}{2}\right) \hat{\mathbf{x}} - b\left(y_{24} - \frac{1}{2}\right) \hat{\mathbf{y}} - c\left(z_{24} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{145} &= -x_{24} \mathbf{a}_1 - y_{24} \mathbf{a}_2 - z_{24} \mathbf{a}_3 = -ax_{24} \hat{\mathbf{x}} - by_{24} \hat{\mathbf{y}} - cz_{24} \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{146} &= \left(x_{24} + \frac{1}{2}\right) \mathbf{a}_1 + y_{24} \mathbf{a}_2 - \left(z_{24} - \frac{1}{2}\right) \mathbf{a}_3 = a\left(x_{24} + \frac{1}{2}\right) \hat{\mathbf{x}} + by_{24} \hat{\mathbf{y}} - c\left(z_{24} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{147} &= x_{24} \mathbf{a}_1 - \left(y_{24} - \frac{1}{2}\right) \mathbf{a}_2 + z_{24} \mathbf{a}_3 = ax_{24} \hat{\mathbf{x}} - b\left(y_{24} - \frac{1}{2}\right) \hat{\mathbf{y}} + cz_{24} \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{148} &= -\left(x_{24} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{24} + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{24} + \frac{1}{2}\right) \mathbf{a}_3 = -a\left(x_{24} - \frac{1}{2}\right) \hat{\mathbf{x}} + b\left(y_{24} + \frac{1}{2}\right) \hat{\mathbf{y}} + c\left(z_{24} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn VIII} \\
\mathbf{B}_{149} &= x_{25} \mathbf{a}_1 + y_{25} \mathbf{a}_2 + z_{25} \mathbf{a}_3 = ax_{25} \hat{\mathbf{x}} + by_{25} \hat{\mathbf{y}} + cz_{25} \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{150} &= -\left(x_{25} - \frac{1}{2}\right) \mathbf{a}_1 - y_{25} \mathbf{a}_2 + \left(z_{25} + \frac{1}{2}\right) \mathbf{a}_3 = -a\left(x_{25} - \frac{1}{2}\right) \hat{\mathbf{x}} - by_{25} \hat{\mathbf{y}} + c\left(z_{25} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{151} &= -x_{25} \mathbf{a}_1 + \left(y_{25} + \frac{1}{2}\right) \mathbf{a}_2 - z_{25} \mathbf{a}_3 = -ax_{25} \hat{\mathbf{x}} + b\left(y_{25} + \frac{1}{2}\right) \hat{\mathbf{y}} - cz_{25} \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{152} &= \left(x_{25} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{25} - \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{25} - \frac{1}{2}\right) \mathbf{a}_3 = a\left(x_{25} + \frac{1}{2}\right) \hat{\mathbf{x}} - b\left(y_{25} - \frac{1}{2}\right) \hat{\mathbf{y}} - c\left(z_{25} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{153} &= -x_{25} \mathbf{a}_1 - y_{25} \mathbf{a}_2 - z_{25} \mathbf{a}_3 = -ax_{25} \hat{\mathbf{x}} - by_{25} \hat{\mathbf{y}} - cz_{25} \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{154} &= \left(x_{25} + \frac{1}{2}\right) \mathbf{a}_1 + y_{25} \mathbf{a}_2 - \left(z_{25} - \frac{1}{2}\right) \mathbf{a}_3 = a\left(x_{25} + \frac{1}{2}\right) \hat{\mathbf{x}} + by_{25} \hat{\mathbf{y}} - c\left(z_{25} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{155} &= x_{25} \mathbf{a}_1 - \left(y_{25} - \frac{1}{2}\right) \mathbf{a}_2 + z_{25} \mathbf{a}_3 = ax_{25} \hat{\mathbf{x}} - b\left(y_{25} - \frac{1}{2}\right) \hat{\mathbf{y}} + cz_{25} \hat{\mathbf{z}} & (8d) & \text{Mn IX} \\
\mathbf{B}_{156} &= -\left(x_{25} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{25} + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{25} + \frac{1}{2}\right) \mathbf{a}_3 = -a\left(x_{25} - \frac{1}{2}\right) \hat{\mathbf{x}} + b\left(y_{25} + \frac{1}{2}\right) \hat{\mathbf{y}} + c\left(z_{25} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8d) & \text{Mn IX}
\end{aligned}$$

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