

α -AlB₁₂ Structure:

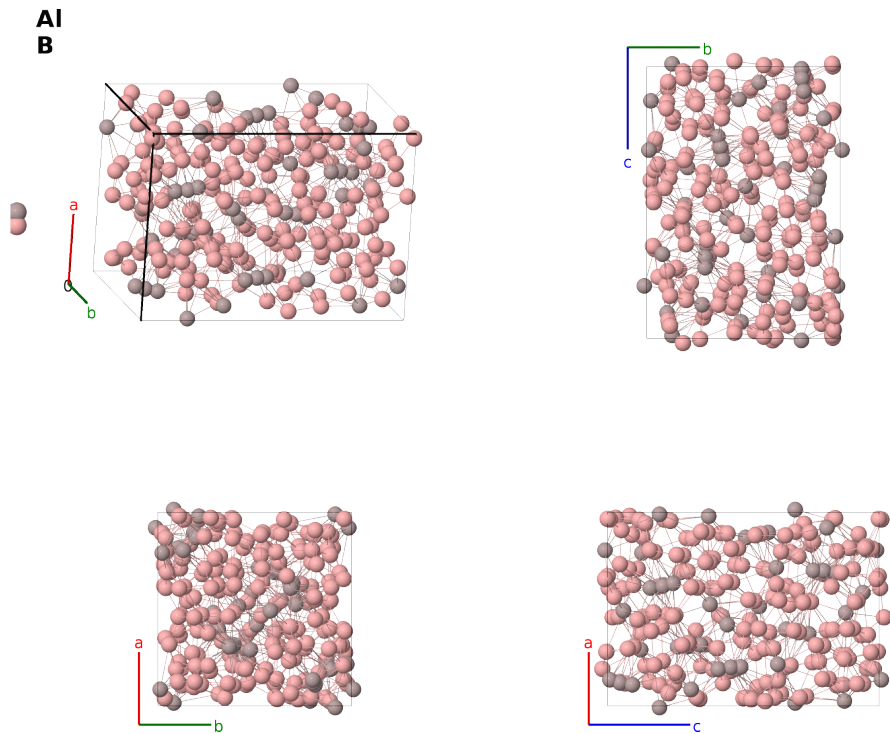
A5B22_tP216_96_5b_2a21b-001

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- D. Hicks, M. J. Mehl, M. Esters, C. Osos, O. Levy, G. L. W. Hart, C. Toher, and S. Curtarolo, *The AFLOW Library of Crystallographic Prototypes: Part 3*, Comput. Mater. Sci. **199**, 110450 (2021), doi: 10.1016/j.commatsci.2021.110450.

<https://aflow.org/prototype-encyclopedia/ZF6C>

https://aflow.org/prototype-encyclopedia/A5B22_tP216_96_5b_2a21b-001

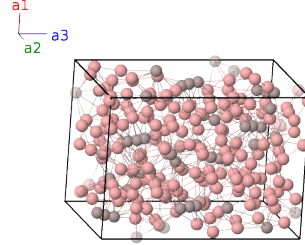


Prototype	AlB ₁₂
AFLOW prototype label	A5B22_tP216_96_5b_2a21b-001
ICSD	1091
CCDC	1591281
Pearson symbol	tP216
Space group number	96
Space group symbol	$P4_32_12$
AFLOW prototype command	<pre>aflow --proto=A5B22_tP216_96_5b_2a21b-001 --params=a, c/a, x₁, x₂, x₃, y₃, z₃, x₄, y₄, z₄, x₅, y₅, z₅, x₆, y₆, z₆, x₇, y₇, z₇, x₈, y₈, z₈, x₉, y₉, z₉, x₁₀, y₁₀, z₁₀, x₁₁, y₁₁, z₁₁, x₁₂, y₁₂, z₁₂, x₁₃, y₁₃, z₁₃, x₁₄, y₁₄, z₁₄, x₁₅, y₁₅, z₁₅, x₁₆, y₁₆, z₁₆, x₁₇, y₁₇, z₁₇, x₁₈, y₁₈, z₁₈, x₁₉, y₁₉, z₁₉, x₂₀, y₂₀, z₂₀, x₂₁, y₂₁, z₂₁, x₂₂, y₂₂, z₂₂, x₂₃, y₂₃, z₂₃, x₂₄, y₂₄, z₂₄, x₂₅, y₂₅, z₂₅, x₂₆, y₂₆, z₂₆, x₂₇, y₂₇, z₂₇, x₂₈, y₂₈, z₂₈</pre>

- AlB_{12} can also be found as orthorhombic $\gamma\text{-AlB}_{12}$. Unfortunately we do not have a determination of the atomic positions in that system.
- As the γ structure has never been isolated from the α structure, and can be transformed into the α structure but not reconstituted from it, $\alpha\text{-AlB}_{12}$ is the ground state of the system. (Higashi, 2000)
- According to (Higashi, 1977), there are thirteen aluminum atoms in this cell, distributed statistically among the five sites. This gives a stoichiometry of $\text{Al}_{0.88}\text{B}_{12}$. The sites Al-I, Al-III, and Al-V are extremely close together. The combined site is 98% occupied, and only one site is occupied at any one position, so the occupancy of these sites are approximately 33%. Similarly, pairs of Al-II atoms are rather close together, and each pair of sites contains approximately 98%, giving an occupation of 49% for the A-II site. If there are thirteen aluminum atoms in the unit cell, puts the occupation of the Al-IV site at 14.5%.
- This structure can also be found in the enantiomorphic space group $P4_12_12$ #92. We follow the authors and place it in $P4_32_12$ #96.

Simple Tetragonal primitive vectors

$$\begin{aligned}\mathbf{a}_1 &= a \hat{\mathbf{x}} \\ \mathbf{a}_2 &= a \hat{\mathbf{y}} \\ \mathbf{a}_3 &= c \hat{\mathbf{z}}\end{aligned}$$



Basis vectors

	Lattice coordinates		Cartesian coordinates	Wyckoff position	Atom type
$\mathbf{B}_1$	$= x_1 \mathbf{a}_1 + x_1 \mathbf{a}_2$	$=$	$ax_1 \hat{\mathbf{x}} + ax_1 \hat{\mathbf{y}}$	(4a)	B I
$\mathbf{B}_2$	$= -x_1 \mathbf{a}_1 - x_1 \mathbf{a}_2 + \frac{1}{2} \mathbf{a}_3$	$=$	$-ax_1 \hat{\mathbf{x}} - ax_1 \hat{\mathbf{y}} + \frac{1}{2}c \hat{\mathbf{z}}$	(4a)	B I
$\mathbf{B}_3$	$= -\left(x_1 - \frac{1}{2}\right) \mathbf{a}_1 + \left(x_1 + \frac{1}{2}\right) \mathbf{a}_2 + \frac{3}{4} \mathbf{a}_3$	$=$	$-a\left(x_1 - \frac{1}{2}\right) \hat{\mathbf{x}} + a\left(x_1 + \frac{1}{2}\right) \hat{\mathbf{y}} + \frac{3}{4}c \hat{\mathbf{z}}$	(4a)	B I
$\mathbf{B}_4$	$= \left(x_1 + \frac{1}{2}\right) \mathbf{a}_1 - \left(x_1 - \frac{1}{2}\right) \mathbf{a}_2 + \frac{1}{4} \mathbf{a}_3$	$=$	$a\left(x_1 + \frac{1}{2}\right) \hat{\mathbf{x}} - a\left(x_1 - \frac{1}{2}\right) \hat{\mathbf{y}} + \frac{1}{4}c \hat{\mathbf{z}}$	(4a)	B I
$\mathbf{B}_5$	$= x_2 \mathbf{a}_1 + x_2 \mathbf{a}_2$	$=$	$ax_2 \hat{\mathbf{x}} + ax_2 \hat{\mathbf{y}}$	(4a)	B II
$\mathbf{B}_6$	$= -x_2 \mathbf{a}_1 - x_2 \mathbf{a}_2 + \frac{1}{2} \mathbf{a}_3$	$=$	$-ax_2 \hat{\mathbf{x}} - ax_2 \hat{\mathbf{y}} + \frac{1}{2}c \hat{\mathbf{z}}$	(4a)	B II
$\mathbf{B}_7$	$= -\left(x_2 - \frac{1}{2}\right) \mathbf{a}_1 + \left(x_2 + \frac{1}{2}\right) \mathbf{a}_2 + \frac{3}{4} \mathbf{a}_3$	$=$	$-a\left(x_2 - \frac{1}{2}\right) \hat{\mathbf{x}} + a\left(x_2 + \frac{1}{2}\right) \hat{\mathbf{y}} + \frac{3}{4}c \hat{\mathbf{z}}$	(4a)	B II
$\mathbf{B}_8$	$= \left(x_2 + \frac{1}{2}\right) \mathbf{a}_1 - \left(x_2 - \frac{1}{2}\right) \mathbf{a}_2 + \frac{1}{4} \mathbf{a}_3$	$=$	$a\left(x_2 + \frac{1}{2}\right) \hat{\mathbf{x}} - a\left(x_2 - \frac{1}{2}\right) \hat{\mathbf{y}} + \frac{1}{4}c \hat{\mathbf{z}}$	(4a)	B II
$\mathbf{B}_9$	$= x_3 \mathbf{a}_1 + y_3 \mathbf{a}_2 + z_3 \mathbf{a}_3$	$=$	$ax_3 \hat{\mathbf{x}} + ay_3 \hat{\mathbf{y}} + cz_3 \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{10}$	$= -x_3 \mathbf{a}_1 - y_3 \mathbf{a}_2 + \left(z_3 + \frac{1}{2}\right) \mathbf{a}_3$	$=$	$-ax_3 \hat{\mathbf{x}} - ay_3 \hat{\mathbf{y}} + c\left(z_3 + \frac{1}{2}\right) \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{11}$	$= -\left(y_3 - \frac{1}{2}\right) \mathbf{a}_1 + \left(x_3 + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_3 + \frac{3}{4}\right) \mathbf{a}_3$	$=$	$-a\left(y_3 - \frac{1}{2}\right) \hat{\mathbf{x}} + a\left(x_3 + \frac{1}{2}\right) \hat{\mathbf{y}} + c\left(z_3 + \frac{3}{4}\right) \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{12}$	$= \left(y_3 + \frac{1}{2}\right) \mathbf{a}_1 - \left(x_3 - \frac{1}{2}\right) \mathbf{a}_2 + \left(z_3 + \frac{1}{4}\right) \mathbf{a}_3$	$=$	$a\left(y_3 + \frac{1}{2}\right) \hat{\mathbf{x}} - a\left(x_3 - \frac{1}{2}\right) \hat{\mathbf{y}} + c\left(z_3 + \frac{1}{4}\right) \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{13}$	$= -\left(x_3 - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_3 + \frac{1}{2}\right) \mathbf{a}_2 - \left(z_3 - \frac{3}{4}\right) \mathbf{a}_3$	$=$	$-a\left(x_3 - \frac{1}{2}\right) \hat{\mathbf{x}} + a\left(y_3 + \frac{1}{2}\right) \hat{\mathbf{y}} - c\left(z_3 - \frac{3}{4}\right) \hat{\mathbf{z}}$	(8b)	Al I

$\mathbf{B}_{14} =$	$(x_3 + \frac{1}{2}) \mathbf{a}_1 - (y_3 - \frac{1}{2}) \mathbf{a}_2 - (z_3 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_3 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_3 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_3 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{15} =$	$y_3 \mathbf{a}_1 + x_3 \mathbf{a}_2 - z_3 \mathbf{a}_3$	$=$	$ay_3 \hat{\mathbf{x}} + ax_3 \hat{\mathbf{y}} - cz_3 \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{16} =$	$-y_3 \mathbf{a}_1 - x_3 \mathbf{a}_2 - (z_3 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_3 \hat{\mathbf{x}} - ax_3 \hat{\mathbf{y}} - c(z_3 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al I
$\mathbf{B}_{17} =$	$x_4 \mathbf{a}_1 + y_4 \mathbf{a}_2 + z_4 \mathbf{a}_3$	$=$	$ax_4 \hat{\mathbf{x}} + ay_4 \hat{\mathbf{y}} + cz_4 \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{18} =$	$-x_4 \mathbf{a}_1 - y_4 \mathbf{a}_2 + (z_4 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_4 \hat{\mathbf{x}} - ay_4 \hat{\mathbf{y}} + c(z_4 + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{19} =$	$-(y_4 - \frac{1}{2}) \mathbf{a}_1 + (x_4 + \frac{1}{2}) \mathbf{a}_2 + (z_4 + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_4 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_4 + \frac{1}{2}) \hat{\mathbf{y}} + c(z_4 + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{20} =$	$(y_4 + \frac{1}{2}) \mathbf{a}_1 - (x_4 - \frac{1}{2}) \mathbf{a}_2 + (z_4 + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_4 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_4 - \frac{1}{2}) \hat{\mathbf{y}} + c(z_4 + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{21} =$	$-(x_4 - \frac{1}{2}) \mathbf{a}_1 + (y_4 + \frac{1}{2}) \mathbf{a}_2 - (z_4 - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_4 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_4 + \frac{1}{2}) \hat{\mathbf{y}} - c(z_4 - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{22} =$	$(x_4 + \frac{1}{2}) \mathbf{a}_1 - (y_4 - \frac{1}{2}) \mathbf{a}_2 - (z_4 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_4 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_4 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_4 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{23} =$	$y_4 \mathbf{a}_1 + x_4 \mathbf{a}_2 - z_4 \mathbf{a}_3$	$=$	$ay_4 \hat{\mathbf{x}} + ax_4 \hat{\mathbf{y}} - cz_4 \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{24} =$	$-y_4 \mathbf{a}_1 - x_4 \mathbf{a}_2 - (z_4 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_4 \hat{\mathbf{x}} - ax_4 \hat{\mathbf{y}} - c(z_4 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al II
$\mathbf{B}_{25} =$	$x_5 \mathbf{a}_1 + y_5 \mathbf{a}_2 + z_5 \mathbf{a}_3$	$=$	$ax_5 \hat{\mathbf{x}} + ay_5 \hat{\mathbf{y}} + cz_5 \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{26} =$	$-x_5 \mathbf{a}_1 - y_5 \mathbf{a}_2 + (z_5 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_5 \hat{\mathbf{x}} - ay_5 \hat{\mathbf{y}} + c(z_5 + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{27} =$	$-(y_5 - \frac{1}{2}) \mathbf{a}_1 + (x_5 + \frac{1}{2}) \mathbf{a}_2 + (z_5 + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_5 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_5 + \frac{1}{2}) \hat{\mathbf{y}} + c(z_5 + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{28} =$	$(y_5 + \frac{1}{2}) \mathbf{a}_1 - (x_5 - \frac{1}{2}) \mathbf{a}_2 + (z_5 + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_5 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_5 - \frac{1}{2}) \hat{\mathbf{y}} + c(z_5 + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{29} =$	$-(x_5 - \frac{1}{2}) \mathbf{a}_1 + (y_5 + \frac{1}{2}) \mathbf{a}_2 - (z_5 - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_5 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_5 + \frac{1}{2}) \hat{\mathbf{y}} - c(z_5 - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{30} =$	$(x_5 + \frac{1}{2}) \mathbf{a}_1 - (y_5 - \frac{1}{2}) \mathbf{a}_2 - (z_5 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_5 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_5 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_5 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{31} =$	$y_5 \mathbf{a}_1 + x_5 \mathbf{a}_2 - z_5 \mathbf{a}_3$	$=$	$ay_5 \hat{\mathbf{x}} + ax_5 \hat{\mathbf{y}} - cz_5 \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{32} =$	$-y_5 \mathbf{a}_1 - x_5 \mathbf{a}_2 - (z_5 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_5 \hat{\mathbf{x}} - ax_5 \hat{\mathbf{y}} - c(z_5 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al III
$\mathbf{B}_{33} =$	$x_6 \mathbf{a}_1 + y_6 \mathbf{a}_2 + z_6 \mathbf{a}_3$	$=$	$ax_6 \hat{\mathbf{x}} + ay_6 \hat{\mathbf{y}} + cz_6 \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{34} =$	$-x_6 \mathbf{a}_1 - y_6 \mathbf{a}_2 + (z_6 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_6 \hat{\mathbf{x}} - ay_6 \hat{\mathbf{y}} + c(z_6 + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{35} =$	$-(y_6 - \frac{1}{2}) \mathbf{a}_1 + (x_6 + \frac{1}{2}) \mathbf{a}_2 + (z_6 + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_6 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_6 + \frac{1}{2}) \hat{\mathbf{y}} + c(z_6 + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{36} =$	$(y_6 + \frac{1}{2}) \mathbf{a}_1 - (x_6 - \frac{1}{2}) \mathbf{a}_2 + (z_6 + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_6 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_6 - \frac{1}{2}) \hat{\mathbf{y}} + c(z_6 + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{37} =$	$-(x_6 - \frac{1}{2}) \mathbf{a}_1 + (y_6 + \frac{1}{2}) \mathbf{a}_2 - (z_6 - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_6 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_6 + \frac{1}{2}) \hat{\mathbf{y}} - c(z_6 - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{38} =$	$(x_6 + \frac{1}{2}) \mathbf{a}_1 - (y_6 - \frac{1}{2}) \mathbf{a}_2 - (z_6 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_6 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_6 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_6 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{39} =$	$y_6 \mathbf{a}_1 + x_6 \mathbf{a}_2 - z_6 \mathbf{a}_3$	$=$	$ay_6 \hat{\mathbf{x}} + ax_6 \hat{\mathbf{y}} - cz_6 \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{40} =$	$-y_6 \mathbf{a}_1 - x_6 \mathbf{a}_2 - (z_6 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_6 \hat{\mathbf{x}} - ax_6 \hat{\mathbf{y}} - c(z_6 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al IV
$\mathbf{B}_{41} =$	$x_7 \mathbf{a}_1 + y_7 \mathbf{a}_2 + z_7 \mathbf{a}_3$	$=$	$ax_7 \hat{\mathbf{x}} + ay_7 \hat{\mathbf{y}} + cz_7 \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{42} =$	$-x_7 \mathbf{a}_1 - y_7 \mathbf{a}_2 + (z_7 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_7 \hat{\mathbf{x}} - ay_7 \hat{\mathbf{y}} + c(z_7 + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{43} =$	$-(y_7 - \frac{1}{2}) \mathbf{a}_1 + (x_7 + \frac{1}{2}) \mathbf{a}_2 + (z_7 + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_7 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_7 + \frac{1}{2}) \hat{\mathbf{y}} + c(z_7 + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al V

$\mathbf{B}_{44} =$	$(y_7 + \frac{1}{2}) \mathbf{a}_1 - (x_7 - \frac{1}{2}) \mathbf{a}_2 + (z_7 + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_7 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_7 - \frac{1}{2}) \hat{\mathbf{y}} + c(z_7 + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{45} =$	$-(x_7 - \frac{1}{2}) \mathbf{a}_1 + (y_7 + \frac{1}{2}) \mathbf{a}_2 - (z_7 - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_7 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_7 + \frac{1}{2}) \hat{\mathbf{y}} - c(z_7 - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{46} =$	$(x_7 + \frac{1}{2}) \mathbf{a}_1 - (y_7 - \frac{1}{2}) \mathbf{a}_2 - (z_7 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_7 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_7 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_7 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{47} =$	$y_7 \mathbf{a}_1 + x_7 \mathbf{a}_2 - z_7 \mathbf{a}_3$	$=$	$ay_7 \hat{\mathbf{x}} + ax_7 \hat{\mathbf{y}} - cz_7 \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{48} =$	$-y_7 \mathbf{a}_1 - x_7 \mathbf{a}_2 - (z_7 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_7 \hat{\mathbf{x}} - ax_7 \hat{\mathbf{y}} - c(z_7 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	Al V
$\mathbf{B}_{49} =$	$x_8 \mathbf{a}_1 + y_8 \mathbf{a}_2 + z_8 \mathbf{a}_3$	$=$	$ax_8 \hat{\mathbf{x}} + ay_8 \hat{\mathbf{y}} + cz_8 \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{50} =$	$-x_8 \mathbf{a}_1 - y_8 \mathbf{a}_2 + (z_8 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_8 \hat{\mathbf{x}} - ay_8 \hat{\mathbf{y}} + c(z_8 + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{51} =$	$-(y_8 - \frac{1}{2}) \mathbf{a}_1 + (x_8 + \frac{1}{2}) \mathbf{a}_2 + (z_8 + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_8 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_8 + \frac{1}{2}) \hat{\mathbf{y}} + c(z_8 + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{52} =$	$(y_8 + \frac{1}{2}) \mathbf{a}_1 - (x_8 - \frac{1}{2}) \mathbf{a}_2 + (z_8 + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_8 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_8 - \frac{1}{2}) \hat{\mathbf{y}} + c(z_8 + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{53} =$	$-(x_8 - \frac{1}{2}) \mathbf{a}_1 + (y_8 + \frac{1}{2}) \mathbf{a}_2 - (z_8 - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_8 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_8 + \frac{1}{2}) \hat{\mathbf{y}} - c(z_8 - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{54} =$	$(x_8 + \frac{1}{2}) \mathbf{a}_1 - (y_8 - \frac{1}{2}) \mathbf{a}_2 - (z_8 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_8 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_8 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_8 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{55} =$	$y_8 \mathbf{a}_1 + x_8 \mathbf{a}_2 - z_8 \mathbf{a}_3$	$=$	$ay_8 \hat{\mathbf{x}} + ax_8 \hat{\mathbf{y}} - cz_8 \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{56} =$	$-y_8 \mathbf{a}_1 - x_8 \mathbf{a}_2 - (z_8 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_8 \hat{\mathbf{x}} - ax_8 \hat{\mathbf{y}} - c(z_8 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	B III
$\mathbf{B}_{57} =$	$x_9 \mathbf{a}_1 + y_9 \mathbf{a}_2 + z_9 \mathbf{a}_3$	$=$	$ax_9 \hat{\mathbf{x}} + ay_9 \hat{\mathbf{y}} + cz_9 \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{58} =$	$-x_9 \mathbf{a}_1 - y_9 \mathbf{a}_2 + (z_9 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_9 \hat{\mathbf{x}} - ay_9 \hat{\mathbf{y}} + c(z_9 + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{59} =$	$-(y_9 - \frac{1}{2}) \mathbf{a}_1 + (x_9 + \frac{1}{2}) \mathbf{a}_2 + (z_9 + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_9 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_9 + \frac{1}{2}) \hat{\mathbf{y}} + c(z_9 + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{60} =$	$(y_9 + \frac{1}{2}) \mathbf{a}_1 - (x_9 - \frac{1}{2}) \mathbf{a}_2 + (z_9 + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_9 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_9 - \frac{1}{2}) \hat{\mathbf{y}} + c(z_9 + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{61} =$	$-(x_9 - \frac{1}{2}) \mathbf{a}_1 + (y_9 + \frac{1}{2}) \mathbf{a}_2 - (z_9 - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_9 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_9 + \frac{1}{2}) \hat{\mathbf{y}} - c(z_9 - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{62} =$	$(x_9 + \frac{1}{2}) \mathbf{a}_1 - (y_9 - \frac{1}{2}) \mathbf{a}_2 - (z_9 - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_9 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_9 - \frac{1}{2}) \hat{\mathbf{y}} - c(z_9 - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{63} =$	$y_9 \mathbf{a}_1 + x_9 \mathbf{a}_2 - z_9 \mathbf{a}_3$	$=$	$ay_9 \hat{\mathbf{x}} + ax_9 \hat{\mathbf{y}} - cz_9 \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{64} =$	$-y_9 \mathbf{a}_1 - x_9 \mathbf{a}_2 - (z_9 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_9 \hat{\mathbf{x}} - ax_9 \hat{\mathbf{y}} - c(z_9 - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	B IV
$\mathbf{B}_{65} =$	$x_{10} \mathbf{a}_1 + y_{10} \mathbf{a}_2 + z_{10} \mathbf{a}_3$	$=$	$ax_{10} \hat{\mathbf{x}} + ay_{10} \hat{\mathbf{y}} + cz_{10} \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{66} =$	$-x_{10} \mathbf{a}_1 - y_{10} \mathbf{a}_2 + (z_{10} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{10} \hat{\mathbf{x}} - ay_{10} \hat{\mathbf{y}} + c(z_{10} + \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{67} =$	$-(y_{10} - \frac{1}{2}) \mathbf{a}_1 + (x_{10} + \frac{1}{2}) \mathbf{a}_2 + (z_{10} + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_{10} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{10} + \frac{1}{2}) \hat{\mathbf{y}} + c(z_{10} + \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{68} =$	$(y_{10} + \frac{1}{2}) \mathbf{a}_1 - (x_{10} - \frac{1}{2}) \mathbf{a}_2 + (z_{10} + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_{10} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{10} - \frac{1}{2}) \hat{\mathbf{y}} + c(z_{10} + \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{69} =$	$-(x_{10} - \frac{1}{2}) \mathbf{a}_1 + (y_{10} + \frac{1}{2}) \mathbf{a}_2 - (z_{10} - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_{10} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{10} + \frac{1}{2}) \hat{\mathbf{y}} - c(z_{10} - \frac{3}{4}) \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{70} =$	$(x_{10} + \frac{1}{2}) \mathbf{a}_1 - (y_{10} - \frac{1}{2}) \mathbf{a}_2 - (z_{10} - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_{10} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{10} - \frac{1}{2}) \hat{\mathbf{y}} - c(z_{10} - \frac{1}{4}) \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{71} =$	$y_{10} \mathbf{a}_1 + x_{10} \mathbf{a}_2 - z_{10} \mathbf{a}_3$	$=$	$ay_{10} \hat{\mathbf{x}} + ax_{10} \hat{\mathbf{y}} - cz_{10} \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{72} =$	$-y_{10} \mathbf{a}_1 - x_{10} \mathbf{a}_2 - (z_{10} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{10} \hat{\mathbf{x}} - ax_{10} \hat{\mathbf{y}} - c(z_{10} - \frac{1}{2}) \hat{\mathbf{z}}$	(8b)	B V
$\mathbf{B}_{73} =$	$x_{11} \mathbf{a}_1 + y_{11} \mathbf{a}_2 + z_{11} \mathbf{a}_3$	$=$	$ax_{11} \hat{\mathbf{x}} + ay_{11} \hat{\mathbf{y}} + cz_{11} \hat{\mathbf{z}}$	(8b)	B VI

$\mathbf{B}_{103} =$	$y_{14} \mathbf{a}_1 + x_{14} \mathbf{a}_2 - z_{14} \mathbf{a}_3$	$=$	$ay_{14} \hat{\mathbf{x}} + ax_{14} \hat{\mathbf{y}} - cz_{14} \hat{\mathbf{z}}$	$(8b)$	B IX
$\mathbf{B}_{104} =$	$-y_{14} \mathbf{a}_1 - x_{14} \mathbf{a}_2 - (z_{14} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{14} \hat{\mathbf{x}} - ax_{14} \hat{\mathbf{y}} - c(z_{14} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B IX
$\mathbf{B}_{105} =$	$x_{15} \mathbf{a}_1 + y_{15} \mathbf{a}_2 + z_{15} \mathbf{a}_3$	$=$	$ax_{15} \hat{\mathbf{x}} + ay_{15} \hat{\mathbf{y}} + cz_{15} \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{106} =$	$-x_{15} \mathbf{a}_1 - y_{15} \mathbf{a}_2 + (z_{15} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{15} \hat{\mathbf{x}} - ay_{15} \hat{\mathbf{y}} + c(z_{15} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{107} =$	$-(y_{15} - \frac{1}{2}) \mathbf{a}_1 + (x_{15} + \frac{1}{2}) \mathbf{a}_2 + (z_{15} + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_{15} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{15} + \frac{1}{2}) \hat{\mathbf{y}} + c(z_{15} + \frac{3}{4}) \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{108} =$	$(y_{15} + \frac{1}{2}) \mathbf{a}_1 - (x_{15} - \frac{1}{2}) \mathbf{a}_2 + (z_{15} + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_{15} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{15} - \frac{1}{2}) \hat{\mathbf{y}} + c(z_{15} + \frac{1}{4}) \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{109} =$	$-(x_{15} - \frac{1}{2}) \mathbf{a}_1 + (y_{15} + \frac{1}{2}) \mathbf{a}_2 - (z_{15} - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_{15} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{15} + \frac{1}{2}) \hat{\mathbf{y}} - c(z_{15} - \frac{3}{4}) \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{110} =$	$(x_{15} + \frac{1}{2}) \mathbf{a}_1 - (y_{15} - \frac{1}{2}) \mathbf{a}_2 - (z_{15} - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_{15} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{15} - \frac{1}{2}) \hat{\mathbf{y}} - c(z_{15} - \frac{1}{4}) \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{111} =$	$y_{15} \mathbf{a}_1 + x_{15} \mathbf{a}_2 - z_{15} \mathbf{a}_3$	$=$	$ay_{15} \hat{\mathbf{x}} + ax_{15} \hat{\mathbf{y}} - cz_{15} \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{112} =$	$-y_{15} \mathbf{a}_1 - x_{15} \mathbf{a}_2 - (z_{15} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{15} \hat{\mathbf{x}} - ax_{15} \hat{\mathbf{y}} - c(z_{15} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B X
$\mathbf{B}_{113} =$	$x_{16} \mathbf{a}_1 + y_{16} \mathbf{a}_2 + z_{16} \mathbf{a}_3$	$=$	$ax_{16} \hat{\mathbf{x}} + ay_{16} \hat{\mathbf{y}} + cz_{16} \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{114} =$	$-x_{16} \mathbf{a}_1 - y_{16} \mathbf{a}_2 + (z_{16} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{16} \hat{\mathbf{x}} - ay_{16} \hat{\mathbf{y}} + c(z_{16} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{115} =$	$-(y_{16} - \frac{1}{2}) \mathbf{a}_1 + (x_{16} + \frac{1}{2}) \mathbf{a}_2 + (z_{16} + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_{16} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{16} + \frac{1}{2}) \hat{\mathbf{y}} + c(z_{16} + \frac{3}{4}) \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{116} =$	$(y_{16} + \frac{1}{2}) \mathbf{a}_1 - (x_{16} - \frac{1}{2}) \mathbf{a}_2 + (z_{16} + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_{16} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{16} - \frac{1}{2}) \hat{\mathbf{y}} + c(z_{16} + \frac{1}{4}) \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{117} =$	$-(x_{16} - \frac{1}{2}) \mathbf{a}_1 + (y_{16} + \frac{1}{2}) \mathbf{a}_2 - (z_{16} - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_{16} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{16} + \frac{1}{2}) \hat{\mathbf{y}} - c(z_{16} - \frac{3}{4}) \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{118} =$	$(x_{16} + \frac{1}{2}) \mathbf{a}_1 - (y_{16} - \frac{1}{2}) \mathbf{a}_2 - (z_{16} - \frac{1}{4}) \mathbf{a}_3$	$=$	$a(x_{16} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{16} - \frac{1}{2}) \hat{\mathbf{y}} - c(z_{16} - \frac{1}{4}) \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{119} =$	$y_{16} \mathbf{a}_1 + x_{16} \mathbf{a}_2 - z_{16} \mathbf{a}_3$	$=$	$ay_{16} \hat{\mathbf{x}} + ax_{16} \hat{\mathbf{y}} - cz_{16} \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{120} =$	$-y_{16} \mathbf{a}_1 - x_{16} \mathbf{a}_2 - (z_{16} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{16} \hat{\mathbf{x}} - ax_{16} \hat{\mathbf{y}} - c(z_{16} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XI
$\mathbf{B}_{121} =$	$x_{17} \mathbf{a}_1 + y_{17} \mathbf{a}_2 + z_{17} \mathbf{a}_3$	$=$	$ax_{17} \hat{\mathbf{x}} + ay_{17} \hat{\mathbf{y}} + cz_{17} \hat{\mathbf{z}}$	$(8b)$	B XII
$\mathbf{B}_{122} =$	$-x_{17} \mathbf{a}_1 - y_{17} \mathbf{a}_2 + (z_{17} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{17} \hat{\mathbf{x}} - ay_{17} \hat{\mathbf{y}} + c(z_{17} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XII
$\mathbf{B}_{123} =$	$-(y_{17} - \frac{1}{2}) \mathbf{a}_1 + (x_{17} + \frac{1}{2}) \mathbf{a}_2 + (z_{17} + \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(y_{17} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{17} + \frac{1}{2}) \hat{\mathbf{y}} + c(z_{17} + \frac{3}{4}) \hat{\mathbf{z}}$	$(8b)$	B XII
$\mathbf{B}_{124} =$	$(y_{17} + \frac{1}{2}) \mathbf{a}_1 - (x_{17} - \frac{1}{2}) \mathbf{a}_2 + (z_{17} + \frac{1}{4}) \mathbf{a}_3$	$=$	$a(y_{17} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{17} - \frac{1}{2}) \hat{\mathbf{y}} + c(z_{17} + \frac{1}{4}) \hat{\mathbf{z}}$	$(8b)$	B XII
$\mathbf{B}_{125} =$	$-(x_{17} - \frac{1}{2}) \mathbf{a}_1 + (y_{17} + \frac{1}{2}) \mathbf{a}_2 - (z_{17} - \frac{3}{4}) \mathbf{a}_3$	$=$	$-a(x_{17} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{17} + \frac{1}{2}) \hat{\mathbf{y}} - c(z_{17} - \frac{3}{4}) \hat{\mathbf{z}}$	$(8b)$	B XII
$\mathbf{B}_{126} =$	$(x_{17} + \frac{1}{2}) \mathbf{a}_1$				

$\mathbf{B}_{133} =$	$-(x_{18} - \frac{1}{2}) \mathbf{a}_1 + (y_{18} + \frac{1}{2}) \mathbf{a}_2 -$	$=$	$-a(x_{18} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{18} + \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XIII
	$(z_{18} - \frac{3}{4}) \mathbf{a}_3$		$c(z_{18} - \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{134} =$	$(x_{18} + \frac{1}{2}) \mathbf{a}_1 - (y_{18} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a(x_{18} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{18} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XIII
	$(z_{18} - \frac{1}{4}) \mathbf{a}_3$		$c(z_{18} - \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{135} =$	$y_{18} \mathbf{a}_1 + x_{18} \mathbf{a}_2 - z_{18} \mathbf{a}_3$	$=$	$ay_{18} \hat{\mathbf{x}} + ax_{18} \hat{\mathbf{y}} - cz_{18} \hat{\mathbf{z}}$	$(8b)$	B XIII
$\mathbf{B}_{136} =$	$-y_{18} \mathbf{a}_1 - x_{18} \mathbf{a}_2 - (z_{18} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{18} \hat{\mathbf{x}} - ax_{18} \hat{\mathbf{y}} - c(z_{18} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XIII
$\mathbf{B}_{137} =$	$x_{19} \mathbf{a}_1 + y_{19} \mathbf{a}_2 + z_{19} \mathbf{a}_3$	$=$	$ax_{19} \hat{\mathbf{x}} + ay_{19} \hat{\mathbf{y}} + cz_{19} \hat{\mathbf{z}}$	$(8b)$	B XIV
$\mathbf{B}_{138} =$	$-x_{19} \mathbf{a}_1 - y_{19} \mathbf{a}_2 + (z_{19} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{19} \hat{\mathbf{x}} - ay_{19} \hat{\mathbf{y}} + c(z_{19} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XIV
$\mathbf{B}_{139} =$	$-(y_{19} - \frac{1}{2}) \mathbf{a}_1 + (x_{19} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a(y_{19} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{19} + \frac{1}{2}) \hat{\mathbf{y}} +$	$(8b)$	B XIV
	$(z_{19} + \frac{3}{4}) \mathbf{a}_3$		$c(z_{19} + \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{140} =$	$(y_{19} + \frac{1}{2}) \mathbf{a}_1 - (x_{19} - \frac{1}{2}) \mathbf{a}_2 +$	$=$	$a(y_{19} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{19} - \frac{1}{2}) \hat{\mathbf{y}} +$	$(8b)$	B XIV
	$(z_{19} + \frac{1}{4}) \mathbf{a}_3$		$c(z_{19} + \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{141} =$	$-(x_{19} - \frac{1}{2}) \mathbf{a}_1 + (y_{19} + \frac{1}{2}) \mathbf{a}_2 -$	$=$	$-a(x_{19} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{19} + \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XIV
	$(z_{19} - \frac{3}{4}) \mathbf{a}_3$		$c(z_{19} - \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{142} =$	$(x_{19} + \frac{1}{2}) \mathbf{a}_1 - (y_{19} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a(x_{19} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{19} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XIV
	$(z_{19} - \frac{1}{4}) \mathbf{a}_3$		$c(z_{19} - \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{143} =$	$y_{19} \mathbf{a}_1 + x_{19} \mathbf{a}_2 - z_{19} \mathbf{a}_3$	$=$	$ay_{19} \hat{\mathbf{x}} + ax_{19} \hat{\mathbf{y}} - cz_{19} \hat{\mathbf{z}}$	$(8b)$	B XIV
$\mathbf{B}_{144} =$	$-y_{19} \mathbf{a}_1 - x_{19} \mathbf{a}_2 - (z_{19} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{19} \hat{\mathbf{x}} - ax_{19} \hat{\mathbf{y}} - c(z_{19} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XIV
$\mathbf{B}_{145} =$	$x_{20} \mathbf{a}_1 + y_{20} \mathbf{a}_2 + z_{20} \mathbf{a}_3$	$=$	$ax_{20} \hat{\mathbf{x}} + ay_{20} \hat{\mathbf{y}} + cz_{20} \hat{\mathbf{z}}$	$(8b)$	B XV
$\mathbf{B}_{146} =$	$-x_{20} \mathbf{a}_1 - y_{20} \mathbf{a}_2 + (z_{20} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{20} \hat{\mathbf{x}} - ay_{20} \hat{\mathbf{y}} + c(z_{20} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XV
$\mathbf{B}_{147} =$	$-(y_{20} - \frac{1}{2}) \mathbf{a}_1 + (x_{20} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a(y_{20} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{20} + \frac{1}{2}) \hat{\mathbf{y}} +$	$(8b)$	B XV
	$(z_{20} + \frac{3}{4}) \mathbf{a}_3$		$c(z_{20} + \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{148} =$	$(y_{20} + \frac{1}{2}) \mathbf{a}_1 - (x_{20} - \frac{1}{2}) \mathbf{a}_2 +$	$=$	$a(y_{20} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{20} - \frac{1}{2}) \hat{\mathbf{y}} +$	$(8b)$	B XV
	$(z_{20} + \frac{1}{4}) \mathbf{a}_3$		$c(z_{20} + \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{149} =$	$-(x_{20} - \frac{1}{2}) \mathbf{a}_1 + (y_{20} + \frac{1}{2}) \mathbf{a}_2 -$	$=$	$-a(x_{20} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{20} + \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XV
	$(z_{20} - \frac{3}{4}) \mathbf{a}_3$		$c(z_{20} - \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{150} =$	$(x_{20} + \frac{1}{2}) \mathbf{a}_1 - (y_{20} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a(x_{20} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{20} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XV
	$(z_{20} - \frac{1}{4}) \mathbf{a}_3$		$c(z_{20} - \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{151} =$	$y_{20} \mathbf{a}_1 + x_{20} \mathbf{a}_2 - z_{20} \mathbf{a}_3$	$=$	$ay_{20} \hat{\mathbf{x}} + ax_{20} \hat{\mathbf{y}} - cz_{20} \hat{\mathbf{z}}$	$(8b)$	B XV
$\mathbf{B}_{152} =$	$-y_{20} \mathbf{a}_1 - x_{20} \mathbf{a}_2 - (z_{20} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{20} \hat{\mathbf{x}} - ax_{20} \hat{\mathbf{y}} - c(z_{20} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XV
$\mathbf{B}_{153} =$	$x_{21} \mathbf{a}_1 + y_{21} \mathbf{a}_2 + z_{21} \mathbf{a}_3$	$=$	$ax_{21} \hat{\mathbf{x}} + ay_{21} \hat{\mathbf{y}} + cz_{21} \hat{\mathbf{z}}$	$(8b)$	B XVI
$\mathbf{B}_{154} =$	$-x_{21} \mathbf{a}_1 - y_{21} \mathbf{a}_2 + (z_{21} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{21} \hat{\mathbf{x}} - ay_{21} \hat{\mathbf{y}} + c(z_{21} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XVI
$\mathbf{B}_{155} =$	$-(y_{21} - \frac{1}{2}) \mathbf{a}_1 + (x_{21} + \frac{1}{2}) \mathbf{a}_2 +$	$=$	$-a(y_{21} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{21} + \frac{1}{2}) \hat{\mathbf{y}} +$	$(8b)$	B XVI
	$(z_{21} + \frac{3}{4}) \mathbf{a}_3$		$c(z_{21} + \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{156} =$	$(y_{21} + \frac{1}{2}) \mathbf{a}_1 - (x_{21} - \frac{1}{2}) \mathbf{a}_2 +$	$=$	$a(y_{21} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{21} - \frac{1}{2}) \hat{\mathbf{y}} +$	$(8b)$	B XVI
	$(z_{21} + \frac{1}{4}) \mathbf{a}_3$		$c(z_{21} + \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{157} =$	$-(x_{21} - \frac{1}{2}) \mathbf{a}_1 + (y_{21} + \frac{1}{2}) \mathbf{a}_2 -$	$=$	$-a(x_{21} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{21} + \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XVI
	$(z_{21} - \frac{3}{4}) \mathbf{a}_3$		$c(z_{21} - \frac{3}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{158} =$	$(x_{21} + \frac{1}{2}) \mathbf{a}_1 - (y_{21} - \frac{1}{2}) \mathbf{a}_2 -$	$=$	$a(x_{21} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{21} - \frac{1}{2}) \hat{\mathbf{y}} -$	$(8b)$	B XVI
	$(z_{21} - \frac{1}{4}) \mathbf{a}_3$		$c(z_{21} - \frac{1}{4}) \hat{\mathbf{z}}$		
$\mathbf{B}_{159} =$	$y_{21} \mathbf{a}_1 + x_{21} \mathbf{a}_2 - z_{21} \mathbf{a}_3$	$=$	$ay_{21} \hat{\mathbf{x}} + ax_{21} \hat{\mathbf{y}} - cz_{21} \hat{\mathbf{z}}$	$(8b)$	B XVI
$\mathbf{B}_{160} =$	$-y_{21} \mathbf{a}_1 - x_{21} \mathbf{a}_2 - (z_{21} - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ay_{21} \hat{\mathbf{x}} - ax_{21} \hat{\mathbf{y}} - c(z_{21} - \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XVI
$\mathbf{B}_{161} =$	$x_{22} \mathbf{a}_1 + y_{22} \mathbf{a}_2 + z_{22} \mathbf{a}_3$	$=$	$ax_{22} \hat{\mathbf{x}} + ay_{22} \hat{\mathbf{y}} + cz_{22} \hat{\mathbf{z}}$	$(8b)$	B XVII
$\mathbf{B}_{162} =$	$-x_{22} \mathbf{a}_1 - y_{22} \mathbf{a}_2 + (z_{22} + \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_{22} \hat{\mathbf{x}} - ay_{22} \hat{\mathbf{y}} + c(z_{22} + \frac{1}{2}) \hat{\mathbf{z}}$	$(8b)$	B XVII

$$\begin{aligned}
\mathbf{B}_{192} &= -y_{25} \mathbf{a}_1 - x_{25} \mathbf{a}_2 - \left(z_{25} - \frac{1}{2}\right) \mathbf{a}_3 = -ay_{25} \hat{\mathbf{x}} - ax_{25} \hat{\mathbf{y}} - c \left(z_{25} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XX} \\
\mathbf{B}_{193} &= x_{26} \mathbf{a}_1 + y_{26} \mathbf{a}_2 + z_{26} \mathbf{a}_3 = ax_{26} \hat{\mathbf{x}} + ay_{26} \hat{\mathbf{y}} + cz_{26} \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{194} &= -x_{26} \mathbf{a}_1 - y_{26} \mathbf{a}_2 + \left(z_{26} + \frac{1}{2}\right) \mathbf{a}_3 = -ax_{26} \hat{\mathbf{x}} - ay_{26} \hat{\mathbf{y}} + c \left(z_{26} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{195} &= -\left(y_{26} - \frac{1}{2}\right) \mathbf{a}_1 + \left(x_{26} + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{26} + \frac{3}{4}\right) \mathbf{a}_3 = -a \left(y_{26} - \frac{1}{2}\right) \hat{\mathbf{x}} + a \left(x_{26} + \frac{1}{2}\right) \hat{\mathbf{y}} + c \left(z_{26} + \frac{3}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{196} &= \left(y_{26} + \frac{1}{2}\right) \mathbf{a}_1 - \left(x_{26} - \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{26} + \frac{1}{4}\right) \mathbf{a}_3 = a \left(y_{26} + \frac{1}{2}\right) \hat{\mathbf{x}} - a \left(x_{26} - \frac{1}{2}\right) \hat{\mathbf{y}} + c \left(z_{26} + \frac{1}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{197} &= -\left(x_{26} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{26} + \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{26} - \frac{3}{4}\right) \mathbf{a}_3 = -a \left(x_{26} - \frac{1}{2}\right) \hat{\mathbf{x}} + a \left(y_{26} + \frac{1}{2}\right) \hat{\mathbf{y}} - c \left(z_{26} - \frac{3}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{198} &= \left(x_{26} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{26} - \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{26} - \frac{1}{4}\right) \mathbf{a}_3 = a \left(x_{26} + \frac{1}{2}\right) \hat{\mathbf{x}} - a \left(y_{26} - \frac{1}{2}\right) \hat{\mathbf{y}} - c \left(z_{26} - \frac{1}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{199} &= y_{26} \mathbf{a}_1 + x_{26} \mathbf{a}_2 - z_{26} \mathbf{a}_3 = ay_{26} \hat{\mathbf{x}} + ax_{26} \hat{\mathbf{y}} - cz_{26} \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{200} &= -y_{26} \mathbf{a}_1 - x_{26} \mathbf{a}_2 - \left(z_{26} - \frac{1}{2}\right) \mathbf{a}_3 = -ay_{26} \hat{\mathbf{x}} - ax_{26} \hat{\mathbf{y}} - c \left(z_{26} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XXI} \\
\mathbf{B}_{201} &= x_{27} \mathbf{a}_1 + y_{27} \mathbf{a}_2 + z_{27} \mathbf{a}_3 = ax_{27} \hat{\mathbf{x}} + ay_{27} \hat{\mathbf{y}} + cz_{27} \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{202} &= -x_{27} \mathbf{a}_1 - y_{27} \mathbf{a}_2 + \left(z_{27} + \frac{1}{2}\right) \mathbf{a}_3 = -ax_{27} \hat{\mathbf{x}} - ay_{27} \hat{\mathbf{y}} + c \left(z_{27} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{203} &= -\left(y_{27} - \frac{1}{2}\right) \mathbf{a}_1 + \left(x_{27} + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{27} + \frac{3}{4}\right) \mathbf{a}_3 = -a \left(y_{27} - \frac{1}{2}\right) \hat{\mathbf{x}} + a \left(x_{27} + \frac{1}{2}\right) \hat{\mathbf{y}} + c \left(z_{27} + \frac{3}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{204} &= \left(y_{27} + \frac{1}{2}\right) \mathbf{a}_1 - \left(x_{27} - \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{27} + \frac{1}{4}\right) \mathbf{a}_3 = a \left(y_{27} + \frac{1}{2}\right) \hat{\mathbf{x}} - a \left(x_{27} - \frac{1}{2}\right) \hat{\mathbf{y}} + c \left(z_{27} + \frac{1}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{205} &= -\left(x_{27} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{27} + \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{27} - \frac{3}{4}\right) \mathbf{a}_3 = -a \left(x_{27} - \frac{1}{2}\right) \hat{\mathbf{x}} + a \left(y_{27} + \frac{1}{2}\right) \hat{\mathbf{y}} - c \left(z_{27} - \frac{3}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{206} &= \left(x_{27} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{27} - \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{27} - \frac{1}{4}\right) \mathbf{a}_3 = a \left(x_{27} + \frac{1}{2}\right) \hat{\mathbf{x}} - a \left(y_{27} - \frac{1}{2}\right) \hat{\mathbf{y}} - c \left(z_{27} - \frac{1}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{207} &= y_{27} \mathbf{a}_1 + x_{27} \mathbf{a}_2 - z_{27} \mathbf{a}_3 = ay_{27} \hat{\mathbf{x}} + ax_{27} \hat{\mathbf{y}} - cz_{27} \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{208} &= -y_{27} \mathbf{a}_1 - x_{27} \mathbf{a}_2 - \left(z_{27} - \frac{1}{2}\right) \mathbf{a}_3 = -ay_{27} \hat{\mathbf{x}} - ax_{27} \hat{\mathbf{y}} - c \left(z_{27} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XXII} \\
\mathbf{B}_{209} &= x_{28} \mathbf{a}_1 + y_{28} \mathbf{a}_2 + z_{28} \mathbf{a}_3 = ax_{28} \hat{\mathbf{x}} + ay_{28} \hat{\mathbf{y}} + cz_{28} \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{210} &= -x_{28} \mathbf{a}_1 - y_{28} \mathbf{a}_2 + \left(z_{28} + \frac{1}{2}\right) \mathbf{a}_3 = -ax_{28} \hat{\mathbf{x}} - ay_{28} \hat{\mathbf{y}} + c \left(z_{28} + \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{211} &= -\left(y_{28} - \frac{1}{2}\right) \mathbf{a}_1 + \left(x_{28} + \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{28} + \frac{3}{4}\right) \mathbf{a}_3 = -a \left(y_{28} - \frac{1}{2}\right) \hat{\mathbf{x}} + a \left(x_{28} + \frac{1}{2}\right) \hat{\mathbf{y}} + c \left(z_{28} + \frac{3}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{212} &= \left(y_{28} + \frac{1}{2}\right) \mathbf{a}_1 - \left(x_{28} - \frac{1}{2}\right) \mathbf{a}_2 + \left(z_{28} + \frac{1}{4}\right) \mathbf{a}_3 = a \left(y_{28} + \frac{1}{2}\right) \hat{\mathbf{x}} - a \left(x_{28} - \frac{1}{2}\right) \hat{\mathbf{y}} + c \left(z_{28} + \frac{1}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{213} &= -\left(x_{28} - \frac{1}{2}\right) \mathbf{a}_1 + \left(y_{28} + \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{28} - \frac{3}{4}\right) \mathbf{a}_3 = -a \left(x_{28} - \frac{1}{2}\right) \hat{\mathbf{x}} + a \left(y_{28} + \frac{1}{2}\right) \hat{\mathbf{y}} - c \left(z_{28} - \frac{3}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{214} &= \left(x_{28} + \frac{1}{2}\right) \mathbf{a}_1 - \left(y_{28} - \frac{1}{2}\right) \mathbf{a}_2 - \left(z_{28} - \frac{1}{4}\right) \mathbf{a}_3 = a \left(x_{28} + \frac{1}{2}\right) \hat{\mathbf{x}} - a \left(y_{28} - \frac{1}{2}\right) \hat{\mathbf{y}} - c \left(z_{28} - \frac{1}{4}\right) \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{215} &= y_{28} \mathbf{a}_1 + x_{28} \mathbf{a}_2 - z_{28} \mathbf{a}_3 = ay_{28} \hat{\mathbf{x}} + ax_{28} \hat{\mathbf{y}} - cz_{28} \hat{\mathbf{z}} & (8b) & \text{B XXIII} \\
\mathbf{B}_{216} &= -y_{28} \mathbf{a}_1 - x_{28} \mathbf{a}_2 - \left(z_{28} - \frac{1}{2}\right) \mathbf{a}_3 = -ay_{28} \hat{\mathbf{x}} - ax_{28} \hat{\mathbf{y}} - c \left(z_{28} - \frac{1}{2}\right) \hat{\mathbf{z}} & (8b) & \text{B XXIII}
\end{aligned}$$

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